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Through reformulating the cold, self-gravitating fermion gas as a Bose condensate by identifying their mutual Thomas-Fermi limits, the dissipationless formation of a heavy neutrino star in gravitational collapse is numerically demonstrated. Such stars offer an alternative to supermassive black holes for the compact dark objects at the centers of galaxies.

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Supermassive neutrino stars, in which gravity is balanced by the degeneracy pressure of cold fermions, have been a subject of speculation for more than three decades [1]. Originally, these objects were proposed as models for dark matter in galactic halos and clusters of galaxies, with neutrino masses in the  $\sim$  eV range. More recently, however, degenerate superstars composed of weakly interacting fermions in the  $\sim$  10 keV range have been suggested as an alternative to the supermassive black holes that are purported to exist at the centers of galaxies [2–6]. In fact it has been shown [4] that such degenerate fermion stars could explain the whole range of supermassive compact dark objects which have been observed so far, with masses ranging from  $10^6$  to  $3 \times 10^9 M_\odot$ , merely assuming that a weakly interacting quasi-stable fermion of mass  $m_f \simeq 15$  keV exists in nature.

As an example, the most massive and violent compact dark object ever observed is located at the center of M87 with a mass  $M \simeq 3.2 \times 10^9 M_\odot$  [7]. Interpreting this as a relativistic fermion star at the Oppenheimer-Volkoff [8] limit yields the fermion mass  $m_f \simeq 15$  keV and a radius  $R = 4.45 R_S \sim 1.5$  light-days [3,4], where  $R_S$  is Schwarzschild radius. In this case there is little difference between the fermion star and black hole scenarios because the last stable orbit around a Schwarzschild black hole is at  $3R_S$  anyway.

Extrapolating this down to the compact dark object at the center of our galaxy [9], with mass  $M \simeq 2.6 \times 10^6 M_\odot$ , which is at the lower limit of the mass range, and  $R \lesssim 20$  light-days, the same fermion mass gives  $R \sim 10^4 R_S$ . Consequent upon the shallow potential inside this fermion star, the spectrum of radiation emitted by accreting baryonic matter is cut off for frequencies larger than  $10^{13}$  Hz [3,5], as is observed in the spectrum of the strong radio source Sgr A\* at the galactic center [10]. This fermion star is also consistent [6] with the observed motion of stars within a projected distance of 10 to 30 light-days of Sgr A\* [9].

Of course, it is well-known that 15 keV lies squarely in

the cosmologically forbidden mass range for stable active neutrinos  $\nu$  [11]. Sterile neutrinos are another matter: as shown by Shi and Fuller [12], in the presence of an initial lepton asymmetry of  $\sim 10^{-3}$ , a sterile neutrino  $\nu_s$  of mass  $m_s \sim 10$  keV is resonantly produced with near closure density,  $\Omega = 1$ . Moreover, the resulting energy spectrum is not thermal but rather cut off so as to approximate a cold degenerate Fermi gas. This model is constrained by astrophysical bounds on  $\nu_s \rightarrow \nu\gamma$  [13], however the allowed parameter space includes  $m_s \simeq 15$  keV contributing  $\Omega_s \simeq 0.3$  as favoured by the BOOMERANG data [14].

The statics of degenerate fermion stars is well understood, being the Oppenheimer-Volkoff equation in the relativistic case [8] or the Lane-Emden equation with polytropic index  $n = 3/2$  in the nonrelativistic limit [15]. Alternatively, because  $R \gg 1/m_f$ , one may understand these as the Thomas-Fermi theory applied to self-gravitating systems. The extension of the Thomas-Fermi theory to finite temperature [16,17] has been used to show that at a certain critical temperature weakly interacting massive fermionic matter undergoes a first-order gravitational phase transition from a diffuse to a clustered state, i.e. a nearly degenerate fermion star. Such studies do not, however, bear on the crucial dynamical question of whether the fermion star can form through gravitational collapse of density fluctuations in an orthodox cosmological setting. Indeed, since collisional damping is negligible, one would expect that only a virialized cloud results [11].

$N$ -body simulations of the collisionless Boltzmann or Vlasov equation evidence a rather different picture: the collapse is followed by a series of bounces with matter expelled at each bounce, leaving behind a condensed object [18]. By Liouville's theorem the Vlasov equation describes an incompressible fluid in phase-space so that it respects a form of the exclusion principle. Hence, these  $N$ -body simulations are effectively fermion simulations. What transpires is that gravity, being attractive, self-organises the phase-space fluid into a high-density/momentum core at the expense of other low-density/momentum regions as seen in the evolution of the spherical Vlasov equation [19].

Much the same behaviour is observed in the formation of mini-boson stars through so-called gravitational cooling [20]. Such a mini-boson star is stable by balancing uncertainty and gravitational pressure. A similar mechanism works in the presence of a quartic self-interaction [21] which dominates over uncertainty pres-

sure resulting in an equilibrium radius  $R \gg 1/m_b$  where  $m_b$  is the boson mass [22]. Hence we have a universal description of the physics underlying the formation process: once the collapse proceeds far enough (uncertainty, interaction or degeneracy) pressure results in a bounce, the outgoing shock wave carrying away the binding energy. The virial argument above is circumvented because the ejected matter invalidates its assumption that there is no flow through the boundary.

In this letter we verify the above picture for the formation of the fermion star from a cold nonequilibrium configuration. The dynamical Thomas-Fermi theory was given long ago by Bloch for the electron gas [23], and amounts to Euler's equations for irrotational flow together with an equation of state  $P = P(\rho)$ . The problem is that, transcribed to the self-gravitating fermion gas, there is the Jeans instability, signalled by an imaginary plasma frequency, and thus short-wavelength shocks must be regulated. The usual remedy is to introduce some small numerical viscosity; however it seems imprudent to draw conclusions based on introducing dissipation into what is fundamentally a dissipationless process. Here we take another, literally conservative approach.

In the Newtonian limit a self-interacting boson star is governed by the Gross-Pitaevskii-like equations

$$i \frac{\partial \psi}{\partial t} = \left[ -\frac{\Delta}{2m_b} + V(|\psi|^2) + m_b \varphi \right] \psi \quad (1a)$$

$$\Delta \varphi = 4\pi G |\psi|^2, \quad (1b)$$

where for convenience we have absorbed the boson mass  $m_b$  in the field. Using the ansatz  $\psi = \sqrt{\rho} \exp(-im_b \theta)$ , we arrive at

$$\frac{\partial \rho}{\partial t} = \vec{\nabla} \cdot (\rho \vec{\nabla} \theta) \quad (2a)$$

$$\frac{\partial \theta}{\partial t} = \frac{(\vec{\nabla} \theta)^2}{2} + \frac{1}{m_b} V(\rho) + \varphi - \frac{1}{2m_b^2 \sqrt{\rho}} \Delta \sqrt{\rho}. \quad (2b)$$

The Thomas-Fermi limit is governed by  $m_b \gg |\vec{\nabla} \rho|/\rho$ . Thus neglecting the last term in (2b), Bloch's equations are recovered, with  $\theta$  being the velocity potential and  $V(\rho)$  given by

$$V(\rho) = m_b \int \frac{dP}{\rho}. \quad (3)$$

For a general polytropic equation of state

$$P(\rho) = K \rho^{1+1/n} \quad (4)$$

eq.(3) yields

$$V(\rho) = (n+1) K m_b \rho^{1/n}. \quad (5)$$

Using the potential (5) and introducing

$$R_* \equiv \frac{\Lambda}{m_b} \equiv \frac{[(n+1)K]^{n/2}}{\sqrt{4\pi G}} \quad (6a)$$

$$M_* = \frac{R_*}{G} \quad (6b)$$

as the length and mass scales, respectively, the substitution

$$\psi = \frac{\Psi}{[(n+1)K]^{n/2}} \quad (7)$$

yields the dimensionless equations

$$\frac{i}{\Lambda} \frac{\partial \Psi}{\partial t} = \left[ -\frac{\Delta}{2\Lambda^2} + \varphi + |\Psi|^{2/n} \right] \Psi \quad (8a)$$

$$\Delta \varphi = |\Psi|^2. \quad (8b)$$

The validity of the Newtonian approximation in the static case requires

$$M/M_* = (4\pi)^{-1} \int d^3r |\Psi|^2 \ll 1, \quad n < 3. \quad (9)$$

For weakly interacting degenerate fermions, the polytropic index is  $n = 3/2$ , and

$$R_* = \left( \frac{9\pi^2}{32g_f^2} \right)^{1/4} \frac{m_{\text{Pl}}}{m_f^2} = 0.2325 \left( \frac{\text{keV}}{m_f} \right)^2 \sqrt{\frac{2}{g_f}} \text{ lyr}, \quad (10a)$$

$$M_* = 1.185 \left( \frac{\text{keV}}{m_f} \right)^2 \sqrt{\frac{2}{g_f}} \times 10^{11} M_\odot, \quad (10b)$$

where  $g_f$  is the spin degeneracy and  $m_{\text{Pl}}$  is the Planck mass. By construction a large but finite  $\Lambda$  allows us to simulate the fermionic problem as a bosonic one through their mutual Thomas-Fermi limits, while providing an explicitly energy conserving way of controlling the shocks. The basic regulating mechanism is the “kinetic” part of (8a) which penalises gradients of order  $\Lambda$ . Of course,  $\Lambda$  must be sufficiently large that this term does not change the static scaling relationship

$$M R^{\left(\frac{3-n}{n-1}\right)} = \text{const} \quad (11)$$

arising from the polytropic equation of state. Our criterion is that the ratio of “kinetic” and “pressure” contributions to the static energy functional should be small, in particular for a Gaussian  $\Psi = \alpha \exp[-(r/\beta)^2]$

$$\frac{T[\Psi]}{P[\Psi]} = \left( 1 + \frac{1}{n} \right)^{3/2} \frac{3}{2 \Lambda^2 \beta^2 \alpha^{2/n}}. \quad (12)$$

For  $n = 3/2$  this is independent of the size  $\beta$  for a given mass, yielding the weak condition  $\Lambda \gg 0.97 (M_*/M)^{1/3}$ .

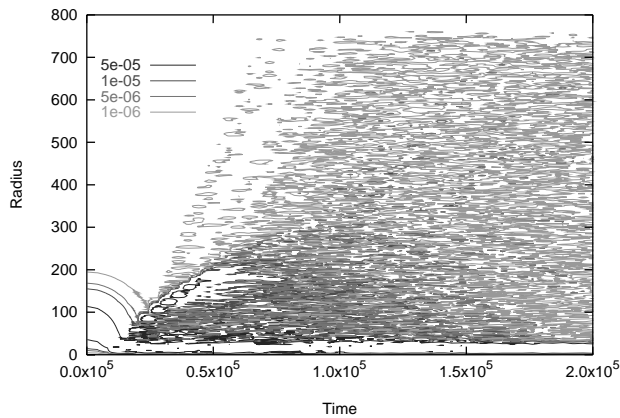


FIG. 1. Contour plot for the evolution of  $|r\Psi|^2$  from the initial configuration in text.

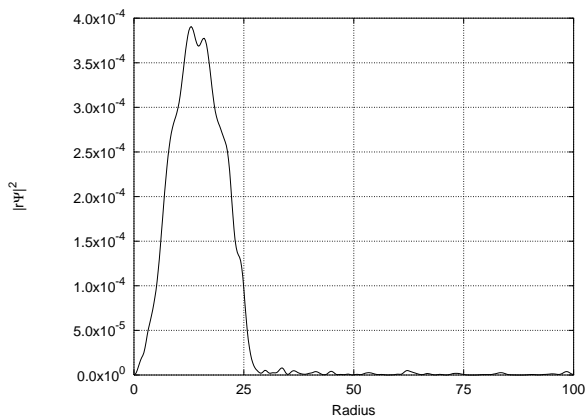


FIG. 2.  $|r\Psi|^2$  versus  $r$  on the final time slice in Figure 1.

In Fig. 1 we display the evolution of  $|r\Psi|^2$  for spherical collapse of a mass  $M = 0.008 M_*$ , initially in the form of a Gaussian with  $\beta = 100 = \Lambda$ . The expected features of bounce and ejection leaving a condensed core are evident. Here we have implemented a velocity dependent imaginary part to the potential in the outer layers of the cavity to remove the ejected fermion matter before it can be artificially reflected by the boundary. The core size  $r/R_* = 26$  and mass  $m/M_* = 0.0057$  is commensurate with a fermion star, however it is far from smooth as evidenced by the plot of  $|r\Psi|^2$  on the final time slice in Fig. 2. This feature may, however, be attributed to the relatively short duration of the simulation.

In summary, using a bosonic representation of the dynamical Thomas-Fermi theory for a self-gravitating gas, we have shown that nonrelativistic, degenerate and weakly interacting fermionic matter will form supermassive fermion stars through gravitational collapse accompanied by ejection. For a fermion mass of  $m_f \simeq 15$  keV

such a superstar is consistent with observations of the compact dark object at the center of our galaxy. A similar demonstration for formation near the Oppenheimer-Volkoff limit, and the question of cosmology with degenerate dark matter requires a general relativistic extension which is under development and will be reported elsewhere.

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